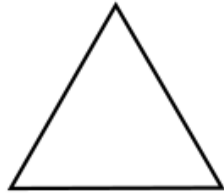


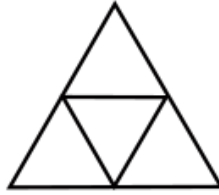
Puzzle of the Week

Filling Triangles with Triangles

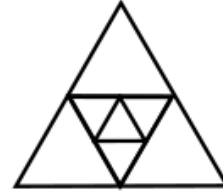
Here's how to fill one large triangle with 1, 4, or 7 triangles..



1

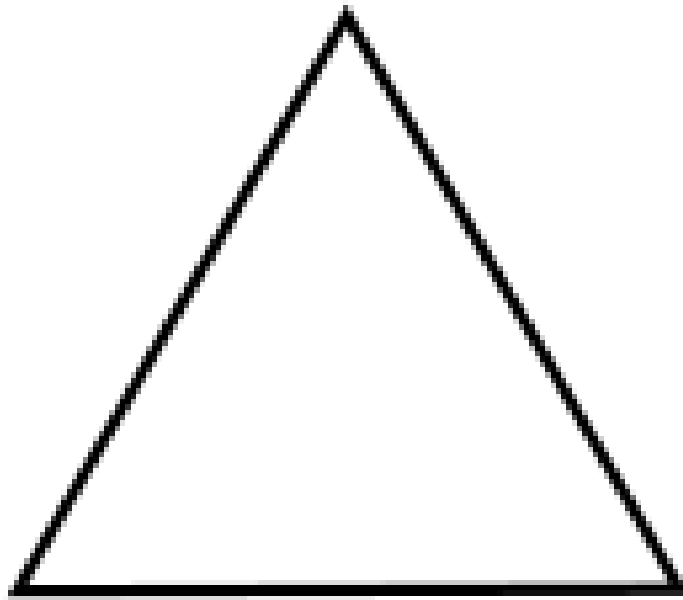


4



7

THE CHALLENGE: Find other triangle counts for filling a large triangle. Can you do it for 2, 3, 5, 6, 8, 9, or 10 triangles?



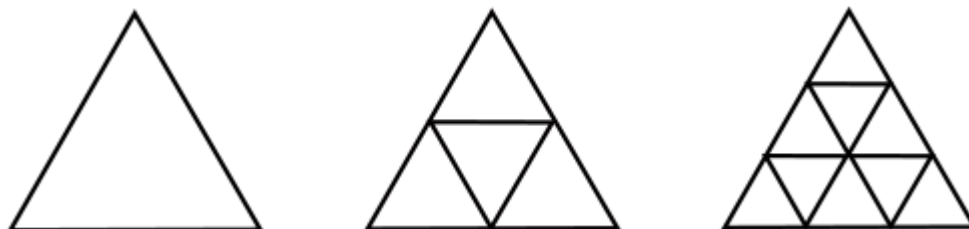
EXPLORATION: When possible, find more than one way to get some of these numbers.

Puzzle of the Week

Filling Triangles w/ Triangles – Notes

THE CHALLENGE: Here is a systematic way of building up various counts.

Squares: A good place to start is with square numbers. It is easy to fill a triangle with 1, 4, and 9 triangles by filling it with the regular pattern used in the following illustration.



Even Numbers: After some experimentation, you can create patterns for 6, 8, 10, or any other larger even number as follows. Start with a large triangle and then put smaller triangles along one side.



Replacing One Triangle: The next big step is to see that any one triangle in a solution can be replaced by any other existing solution. For example, this was done in producing the pattern for 7 in the introduction – the triangle in the center for “4” was replaced with four smaller triangles to produce “7”.

Whenever one triangle is replaced by four triangles, that will increase the total triangle count by 3. Start with the list of solutions using square numbers and even numbers: 1, 4, 6, 8, 9, 10, 12, 14, and 16, and then add 3 to each entry on that list to get 4, 7, 9, 11, 12, 13, 15, and 17. Combining these two lists gives all the possibilities up through 17: 1, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, and 17. Using these ideas, every number above 17 is easy enough, and so we reach the conclusion that:

Answer: Every number is possible except 2, 3, and 5.

EXPLORATION: Some of these numbers can be produced in more than one way. For example, 9 can be done as a 3 by 3 pattern or as 6 plus three more. Of course, there are other interesting ways to fill out a triangle to discover.